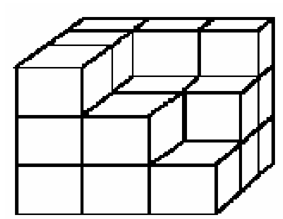
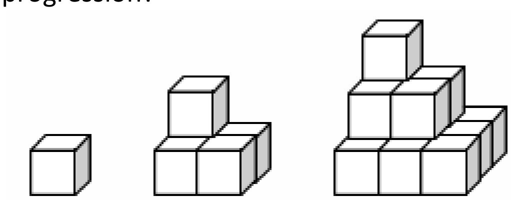
Name: ________________________________ **Three dimensional geometry** *January 28, 2013*

1. The solid figure below began as a cube made up of smaller unit cubes. The only unit cubes that have been removed from the beginning cube are the unit cubes you can see were removed on this front view of the cube. How many unit cubes does this solid figure contain?



2. Below is a progression of towers. How many cubes would be in the fourth tower in this progression?

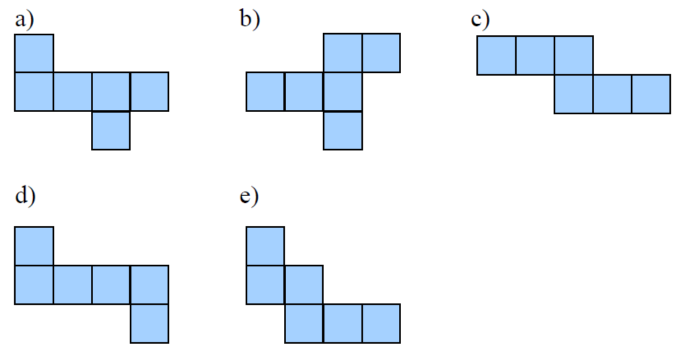


3. Rosie builds a large cube out of sugar cubes. Her large cube has a width of 4 cubes, height of 4 cubes and length of 4 cubes. She wants to add one more layer of cubes all around to make an even larger cube that will have height, width and length 6 cubes. How many additional cubes will she need?

4. A rectangular prism with a square base has a volume of 144 cubic centimeters. If the height is four centimeters, what is the length of the base in centimeters?

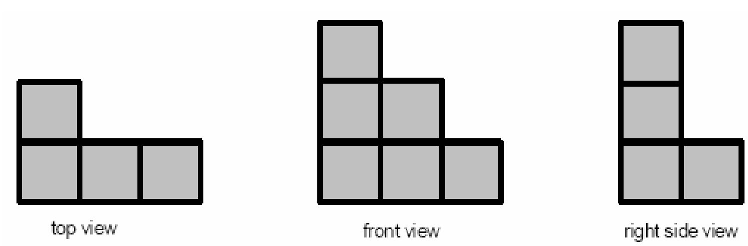
5. A group of toy makers want to wrap the world's biggest toy block with wrapping paper. The block has dimensions of eight feet by six feet by twelve feet. How much wrapping paper, in square feet, will they need in order to wrap the entire toy block? Assume there is no overlap in wrapping paper.

6. Which of the following diagrams cannot be folded into a cube?




7. Toleen's math teacher makes a model of a solid rectangular prism out of white one centimeter cubes to explore surface area and volume. She paints the entire outside surface red. If the dimensions of the prism are 3 cm by 4 cm by 6 cm, how many of the cubes have exactly two sides painted red?

8. A cube sculpture is created from cubes stacked on top of each other and next to each other. Cubes that are stacked meet along their square faces. The top view, front view, and side view of a cube sculpture are given below, where only the squares directly facing you from each view point are shown. What is the least number of cubes you would need to add to the sculpture to make it a solid rectangular prism of cubes?

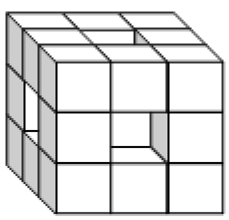


**BONUS PROBLEMS**

9. Anna wants to cover the outside of a rectangular box with colored paper. The box has a square base with area of 16 square inches. The volume of the box is 80 cubic inches. How many square inches of paper will Anna need to completely cover the box, including the top and bottom, with no paper left over?

10. A cubical box without a top is 4 cm on each edge. It contains 64 identical 1 cm cubes that exactly fill the box. How many of these small cubes actually touch the box?

11. Twenty unit cubes are glued together to form this figure, with "holes" which you can see through. The total figure measures 3 x 3 x 3. If the figure is fully dipped in a bucket of paint, how many square units of surface area would be painted?



12. Tape, two centimeters wide, is used to completely cover a cube 10 centimeters on each edge. Find the length of tape needed, in centimeters, if there is no overlap of the tape.

## Solutions

*Note: There are many acceptable strategies to solving each problem. This sheet shows just one strategy.*

1) The full figure without any cubes missing would be 3 x 3 x 3 = 27 cubes. But 5 cubes have been removed (4 from the top layer, and 1 from the middle layer). So there are 22 cubes left.
   **Answer: 22 cubes**

2) The fourth tower will have 4 layers. Counting the layers from bottom to top:

| | |
|---|---|
| $1^{st}$ (bottom) layer: | 16 cubes |
| $2^{nd}$ layer: | 9 cubes |
| $3^{rd}$ layer: | 4 cubes |
| $4^{th}$ (top) layer: | 1 cube |

   Adding up the layers, the fourth tower will have 16+9+4+1 = 30 cubes.
   **Answer: 30 cubes**

3) The original cube has 4x4x4 = 64 sugar cubes.
   The new larger cube has 6x6x6 = 216 sugar cubes.
   Therefore she needs an additional 216 - 64 = 152 sugar cubes to form the new larger cube.
   **Answer: 152 sugar cubes**

4) If the volume is 144 $cm^3$, and the height is 4 cm., the area of the base must be 144 ÷ 4 = 36 $cm^2$.
   Since the base is a square, its length must be $\sqrt{36}$ = 6 cm.
   **Answer: 6 cm.**

5) The block has six faces, which are:

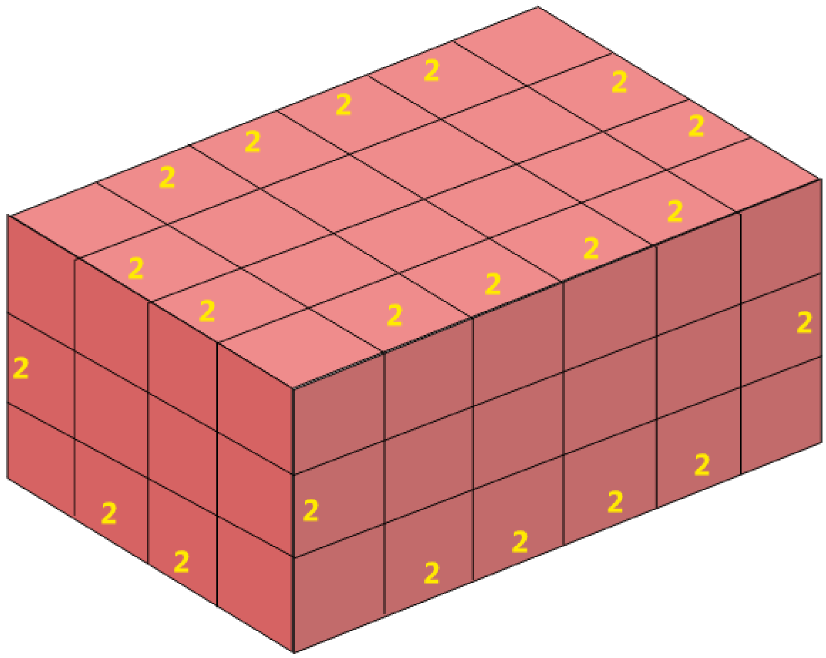
| | | |
|---|---|---|
| Front face: | 8 ft. x 6 ft. | 48 $ft^2$ |
| Back face: | 8 ft. x 6 ft. | 48 $ft^2$ |
| Left face: | 8 ft. x 12 ft. | 96 $ft^2$ |
| Right face: | 8 ft. x 12 ft. | 96 $ft^2$ |
| Top face: | 12 ft. x 6 ft. | 72 $ft^2$ |
| Bottom face: | 12 ft. x 6 ft. | 72 $ft^2$ |

   Adding up the areas of the six faces, we have 48+48+96+96+72+72 = 432 $ft^2$
   **Answer: 432 $ft^2$**

**Solutions (cont.)**

6) If you fold the shapes using real paper or your imagination, you will find that all of them can be folded into a cube except for (E).
**Answer: E**

7) It helps to draw it out:



All of the cubes along the edges of the rectangular solid will have exactly two faces painted red. Don't include the corner cubes, because those have three red faces. All the other cubes that make up this rectangular solid have either one or zero red faces, so don't count those either.

In total, the rectangular solid has 12 edges:

four edges of length 3, each having 1 cube with two red faces.
four edges of length 4, each having 2 cubes with two red faces.
four edges of length 6, each having 4 cubes with two red faces.

So, (4x1) + (4x2) + (4x4) = 28 cubes with two red faces

**Answer: 28 cubes**

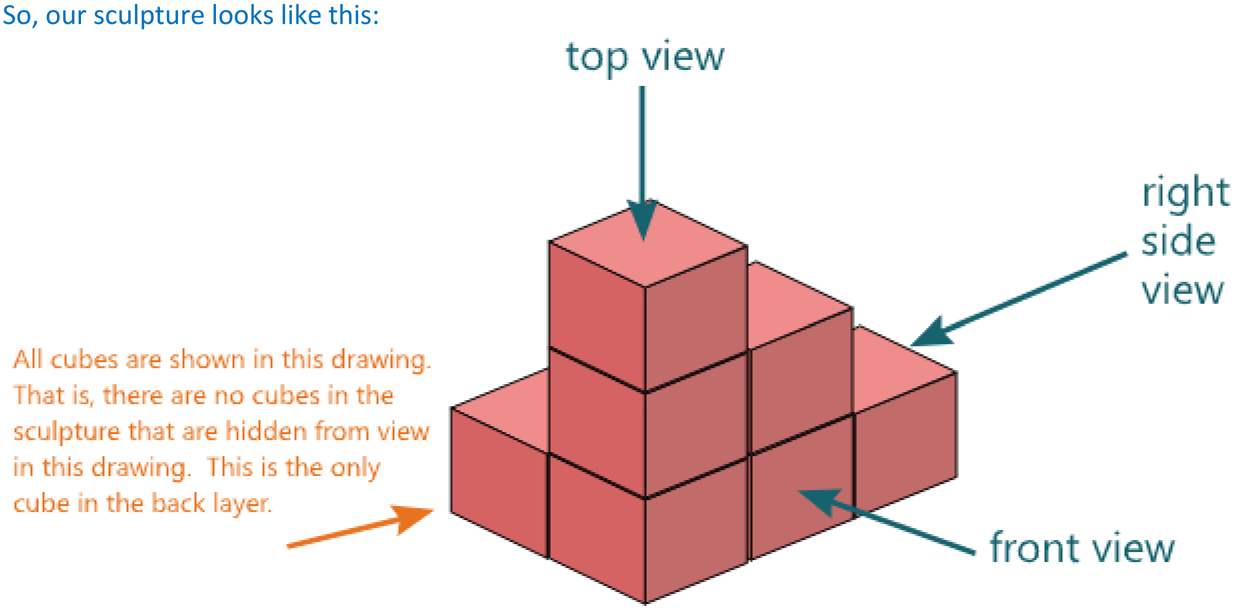
## Solutions (cont.)

8) (This is a very hard problem, but it was given in the $5^{th}$ Grade Math Olympiad competition in 2006.)

The *top view* shows us that there are two layers in this sculpture going from front to back. Furthermore, in the back layer, only one cube is visible from the *top view*. The *right side view* confirms that there are two layers in the sculpture going from front to back. Again, in the back layer, only one cube is visible from the *right side view*. Therefore we can deduce that the back layer truly only contains one single cube. (If the back layer contained two or more cubes, then certainly two or more cubes would be visible from either the *top* or *right side* views.) Furthermore we know that the single cube in the back layer is located in the lower-left corner of the sculpture, as viewed from the front.

Now, looking at the *front view*, we can deduce that the front layer of the sculpture contains exactly the cubes shown in the *front view*. After all, there are only two layers (front to back), and the back layer contains only a single cube in the lower-left corner. Yes, it could theoretically be the case that the front layer was missing its lower-left corner cube, and still the front view would look the same. But then there would be nothing to support the two cubes above it from falling down, due to that little thing called gravity.

So, our sculpture looks like this:




To complete the rectangular solid (3x3x2), we would need to add 3 more cubes to the front layer, and 8 more cubes to the back layer, for a total of 11 more cubes.

**Answer: 11 cubes**

**9)** Find the dimensions of the box. Then add the areas of the faces. The base is a square of area 16 $in^2$ so each of the base's sides is 4 inches long. The given volume of 80 $in^3$ = base x height, so the height is 5 inches. The top and bottom each have an area of 16 $in^2$. Each side has an area of 4 x 5 = 20 $in^2$. The amount of paper needed is 16 + 16 + 20 + 20 + 20 + 20 = 112 $in^2$.

**Answer: 112 $in^2$**

## Solutions (cont.)

**10)** Count the cubes that touch the box in each horizontal layer. In the bottom layer, all 16 cubes touch the box. In each of the three other layers, all the outer cubes touch either one or two sides of the box. That is 12 outer cubes per layer. Thus there are a total of 16 + 3 x 12 = 52 small cubes that touch the bottom or a side of the box.

**Answer: 52 cubes**

**11)** Count in an organized way: exterior vs. interior squares. Each face of the figure has 8 painted squares for a total of 6 x 8 = 48 painted faces. Inside each "hole" 4 squares are painted for a total of 6 x 4 = 24 painted faces. The total number of square units of painted surface is 24 + 48 = 72.

**Answer: 72**

12) A 10-cm cube has a surface area of 100 square centimeters on each of its 6 faces, making a total of 600 $cm^2$ to be covered. To get an area of 600 $cm^2$ of tape which is 2 cm wide, divide 600 $cm^2$ by 2 cm. 600 ÷ 2 = 300 cm of tape.

**Answer: 300 cm of tape**